Joint Decoding and Carrier Synchronization using Factor Graphs

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As outlined already by Wiberg [4], iterative algorithms for joint decoding and channel estimation may be derived from the factor graph [2] of the code and the channel. We report some first results from a systematic study of message passing algorithms for joint decoding and phase estimation. We consider discrete-time channels of the form

$$Y_k = X_k e^{j\Theta_k} + N_k, \qquad (1)$$

where X_k is the (complex-valued) channel input symbol at time $k \in \mathbb{Z}$, Y_k is the corresponding received symbol, Θ_k is the unknown phase, and N_k is white Gaussian noise with (known) variance σ^2. For the sake of definiteness, we will assume that the channel input symbols X_k take values in $\{+1, -1\}$ and are protected by a low-density parity check (LDPC) code.

We will consider three different models for the evolution of the phase Θ_k:

Constant Phase: $\Theta_k = \Theta$, an unknown constant.

Random Walk:

$$\Theta_k = (\Theta_{k-1} + W_k) \bmod 2\pi, \qquad (2)$$

where W_k is white Gaussian noise with known variance σ_W^2.

Random Walk with Unknown Drift:

$$\Theta_k = (\Theta_{k-1} + \Omega + W_k) \bmod 2\pi, \qquad (3)$$

where $\Omega \in [0, 2\pi)$ is unknown and with W_k as above.

Phase estimation is of course an old subject, and several studies of joint iterative decoding and phase estimation have recently appeared, e.g. [1]. In constrast to most of these works, our emphasis here is not on the performance analysis (which must of course be our ultimate goal), but on a systematic investigation of suitable message types and the corresponding message update rules.

The system described in the above is easily translated into the Forney-style factor graph (FFG) of Fig. 1, which represents the factorization of the joint probability density of all variables. The upper part of the graph is the indicator function of the LDPC code, with parity check nodes in the top row that are "randomly" connected to equality constraint nodes ("bit nodes"). The bottom row of the graph represents the factors $p(y_k|z_k) = \gamma e^{-(y_k - z_k)^2/2\sigma^2}$, where $\gamma \in \mathbb{R}$ is a suitable scale factor.

The computation of the messages inside the graph of the LDPC code is standard; we therefore consider only the computation of the messages inside, and out of, the graph of the phase model. We will consider different phase estimators operating with three different message types.

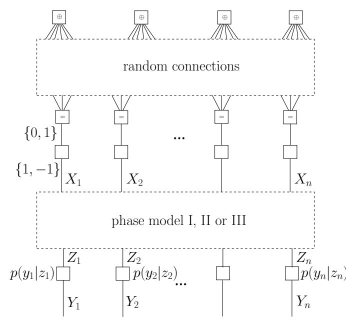

Figure 1: FFG of LDPC code and the channel model.

1. An obvious choice is to quantize the possible values of Θ_k. We implemented the sum-product algorithm using quantized messages for all three phase models.
2. An alternative to quantized messages are mod-2π Gaussian mixtures. We implemented the sum-product algorithm for the three phase models using bimodal Gaussian mixtures.
3. For the constant-phase model, the "upwards" sum-product message $\mu_{\text{up}}(x)$ along some edge X_k can, in fact, be computed in closed form, as a function of the observations y_k.

We will present some preliminary simulation results for all the described message passing receivers and all three channel models. The quantized-messages receiver has the highest computational complexity. From our preliminary simulations, the performance of the three receivers appears to be quite similar.

It is clear that near-capacity performance of such schemes can only be achieved if the code and (the number and the position of) the pilot symbols are optimized jointly.

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