Applications of graphical models in signal processing

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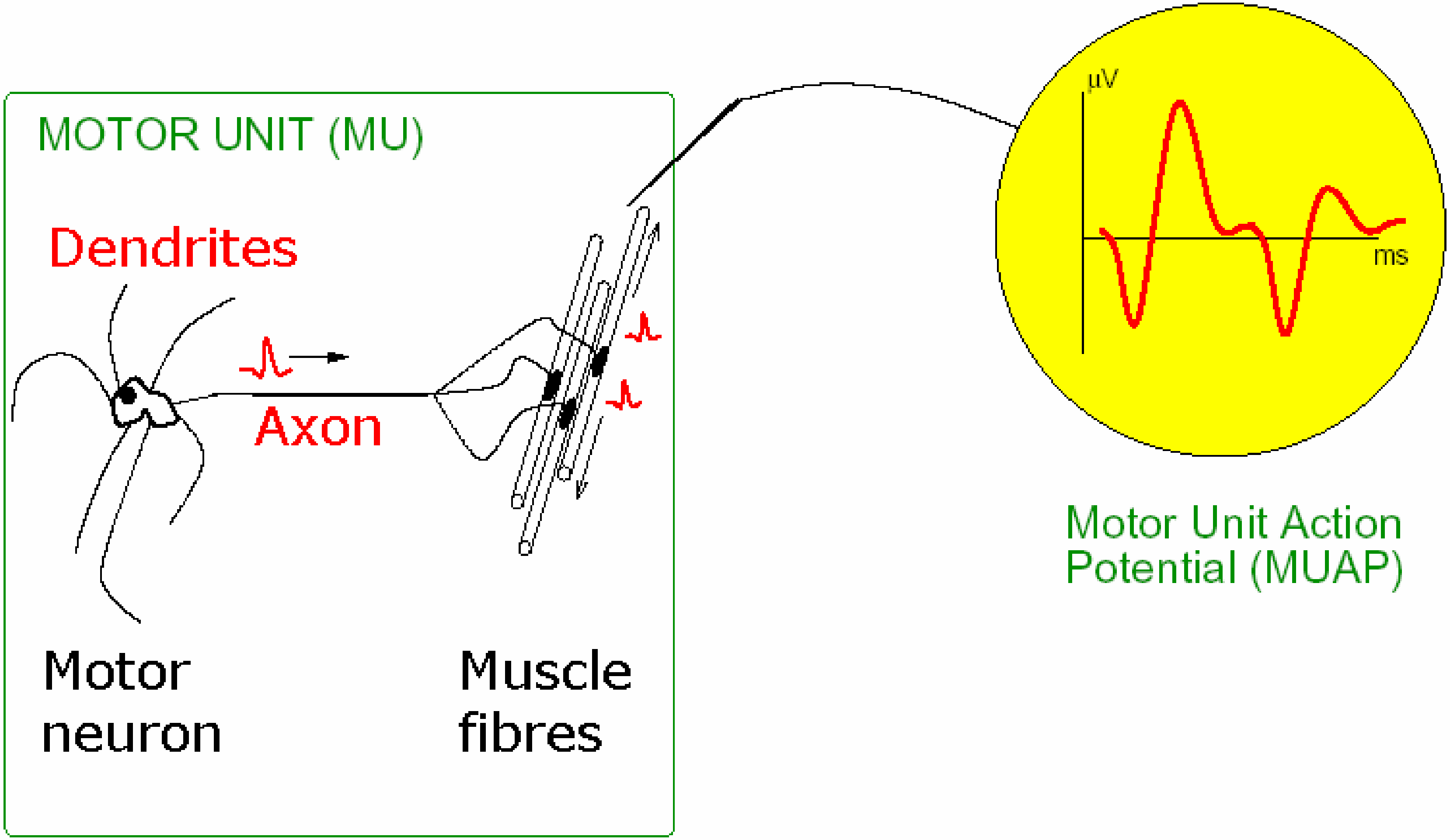
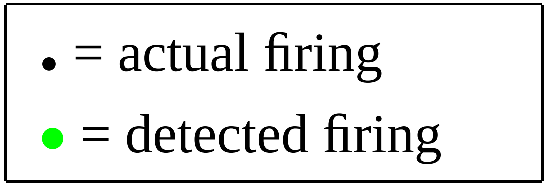
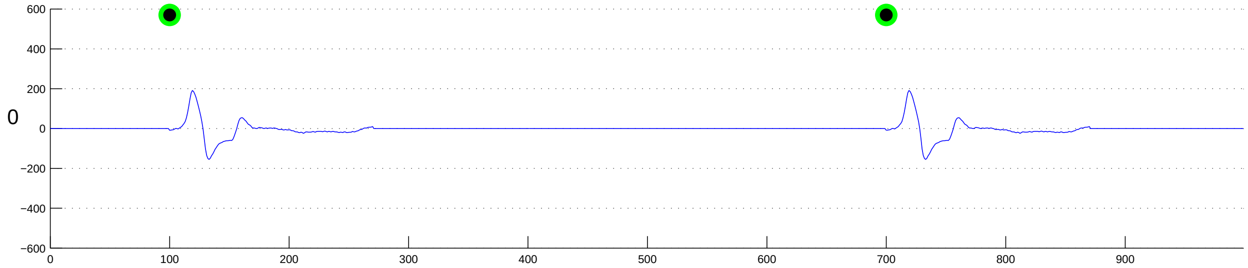
Electromyography



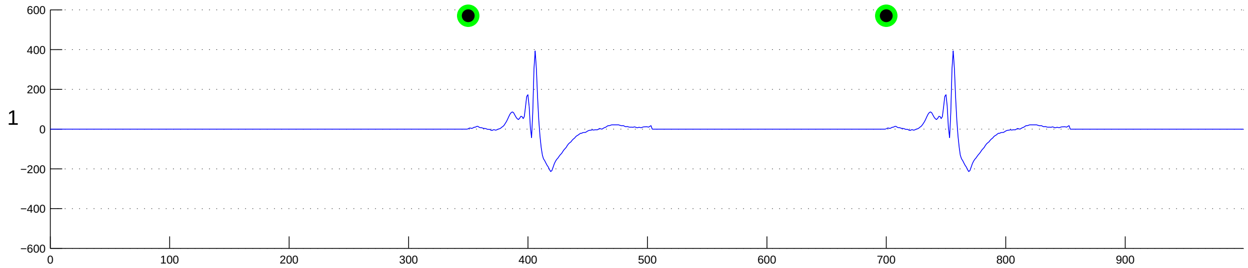
EMG Signal Decomposition



Signal from MU 0:

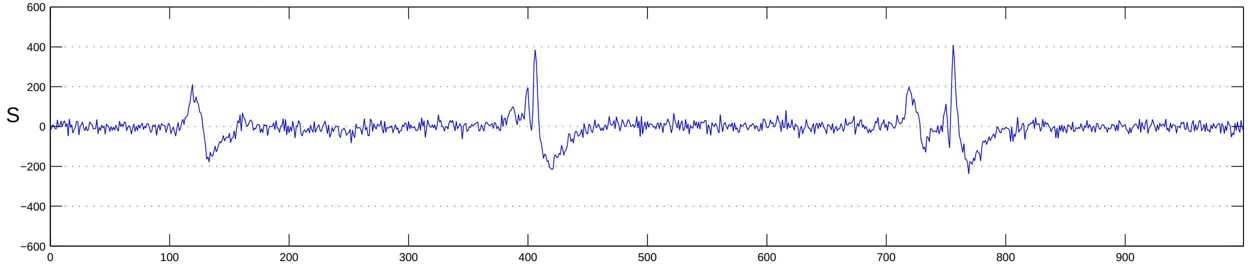


Signal from MU 1:



EMG signal:

$(\sigma_{\text{noise}} = 20)$



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Estimation Theory

Optimal estimators

- Block MAP (Maximum a posteriori)

$$\hat{\mathbf{x}} = \text{argmax}_{\mathbf{x}}\, p(\mathbf{x}|\mathbf{y}) = \text{argmax}_{\mathbf{x}}\, p(\mathbf{x}, \mathbf{y})$$

- Symbol MAP

$$\hat{x}_k = \text{argmax}_{x_k}\, p(x_k|\mathbf{y}) = \text{argmax}_{x_k}\, p(x_k, \mathbf{y}) = \text{argmax}_{x_k} \sum_{\sim x_k} p(\mathbf{x}, \mathbf{y})$$

Key observations

- Summation and maximization are the involved operations.
- The probability function $p(\mathbf{x}, \mathbf{y})$ factorizes in most practical systems . . .
- ... which leads to more efficient computations.
- Despite this fact, optimal estimators are often infeasible . . .
- ... and hence one typically resorts to approximations.

Our methodology

Our approach rests on two pillars

1. A graphical representation of $p(\mathbf{x}, \mathbf{y})$, a so-called factor graph.
2. An algorithm (sum-product algorithm) that computes marginals $p(x_k)$ and operates on a factor graph that represents the system at hand.

Within this framework

- the factorization of $p(\mathbf{x}, \mathbf{y})$ is intensively exploited and, as a consequence, marginals $p(x_k)$ are computed efficiently.
- approximations can be designed in a systematic fashion.

Motivation

- Many algorithms in coding, signal processing, and machine learning may be viewed as instances of the sum-product algorithm.

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- Specific instances of such algorithms include Kalman filtering and smoothing, the forward-backward algorithm for hidden Markov models, Viterbi algorithm, probability propagation in Bayesian networks, decoding algorithms for error correcting codes (in particular iterative decoding of turbo codes and low-density parity check codes), . . .

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- Applications at ISI: Decomposition of EMG signals, signal processing for hearing aids, synchronization in digital communications, decoding, estimation/detection with analog circuits, computation of capacities of communication channels, smart fire detection, body-state estimation, seismosomnography, . . .

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- Factor graphs and message-passing algorithm have been proven to be very powerful in decoding and signal processing.

Overview

- Factor graphs
- Sum-product algorithm
- Application: decomposition of EMG-signals
- Conclusions

Factor Graphs (FGs)

- Factor graphs represent the factorization of a function.
- Example

$$f(x_1, x_2, x_3, x_4, x_5) = f_A(x_1, x_2, x_3) f_B(x_3, x_4, x_5) f_C(x_4).$$

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- x_1, x_2, x_3, x_4, and x_5 are the variables.
- f is called the global function.
- f_A, f_B, and f_C are called local functions.

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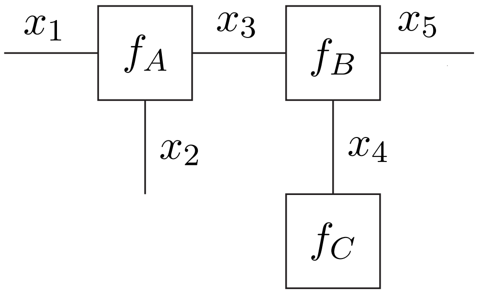
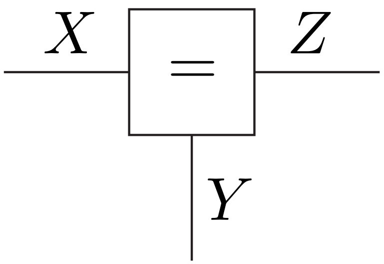


- Rules for drawing a factor graph
  - A node for every factor
  - An edge for every variable
  - Node g is connected to edge x iff variable x appears in factor g

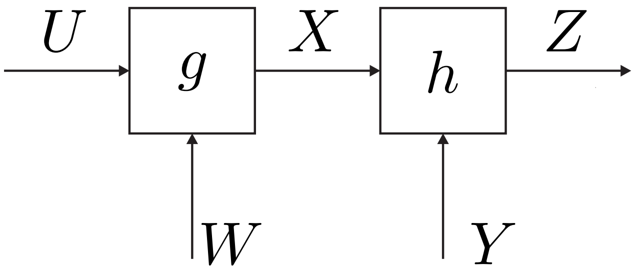
Equality constraint node

$$f_{=}(x, y, z) \triangleq \begin{cases} 1, & \text{if } x = y = z \\ 0, & \text{else} \end{cases}$$

$$= \delta[x - y] \cdot \delta[x - z]$$

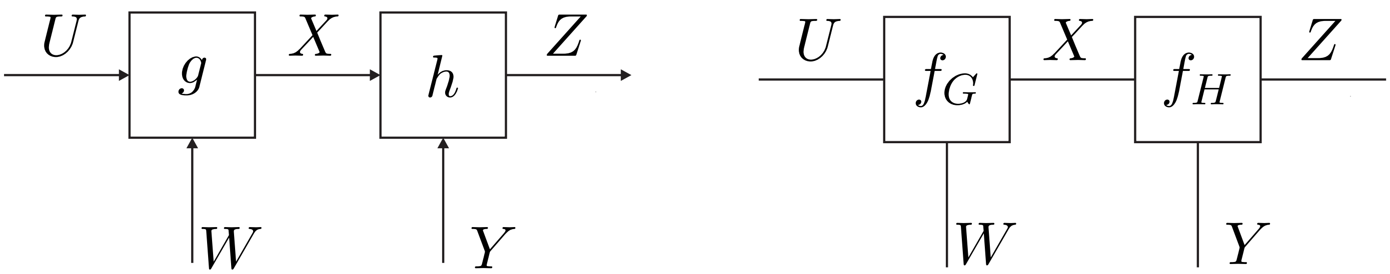


Block Diagrams and Factor Graphs



- Block diagram: $X = g(U, W)$, $Z = h(X, Y)$

Block Diagrams and Factor Graphs



- Block diagram: $X = g(U, W)$, $Z = h(X, Y)$

- Factor graph: $f_G = \delta(x - g(u, w))$, $f_H = \delta(z - h(x, y))$
- Global function $f(u, w, x, y, z) = \delta(x - g(u, w)) \cdot \delta(z - h(x, y))$

- $f \neq 0$ (configuration valid) iff the configuration is consistent with the functions g and h of the block diagram.

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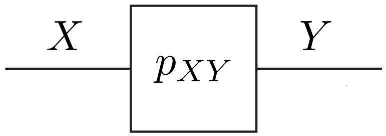
Joint Probability Distribution

- Factor graphs are used mostly to represent probabilistic models.
- Given is a joint probability distribution

$$p_{XY}(x, y)$$

of the two discrete random variables X and Y.

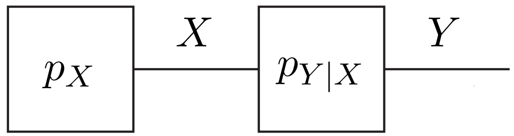


Factorization of Joint Probability Distribution

- Chain rule

$$p_{XY}(x,y) = p_X(x)p_{Y|X}(y|x)$$

- Factor graph

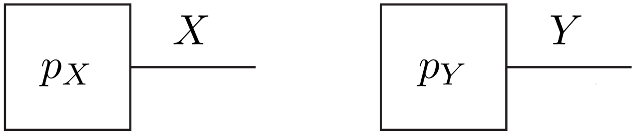


Independent Random Variables

- If the two random variables X and Y are independent, then $p_{Y|X}(y|x) = p_Y(y)$ and we get the factorization

$$p_{XY}(x, y) = p_X(x)p_Y(y).$$

- Factor graph



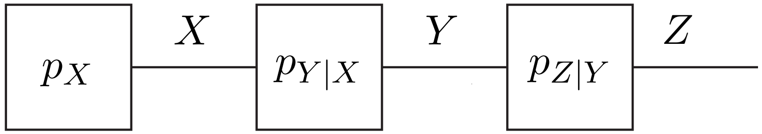
- If two components are unconnected, then every random variable of one component is independent of every random variable in the other component.

Markov Chain

- Markov chain of the form

$$p_{XYZ}(x, y, z) = p_X(x) \cdot p_{Y|X}(y|x) \cdot p_{Z|Y}(z|y)$$

- Factor graph



Summary: Factor Graphs

- Factor graphs
- Equal node
- Block diagrams and factor graphs
- Probabilistic models represented by factor graph

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Introduction (4)

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Computing marginals

- Given: Discrete probability mass function

$$f(x_1, \ldots, x_8) = \Big(f_1(x_1) f_2(x_2) f_3(x_1, x_2, x_3, x_4)\Big) \cdot \Big(f_4(x_4, x_5, x_6) f_5(x_5) \big(f_6(x_6, x_7, x_8) f_7(x_7)\big)\Big)$$

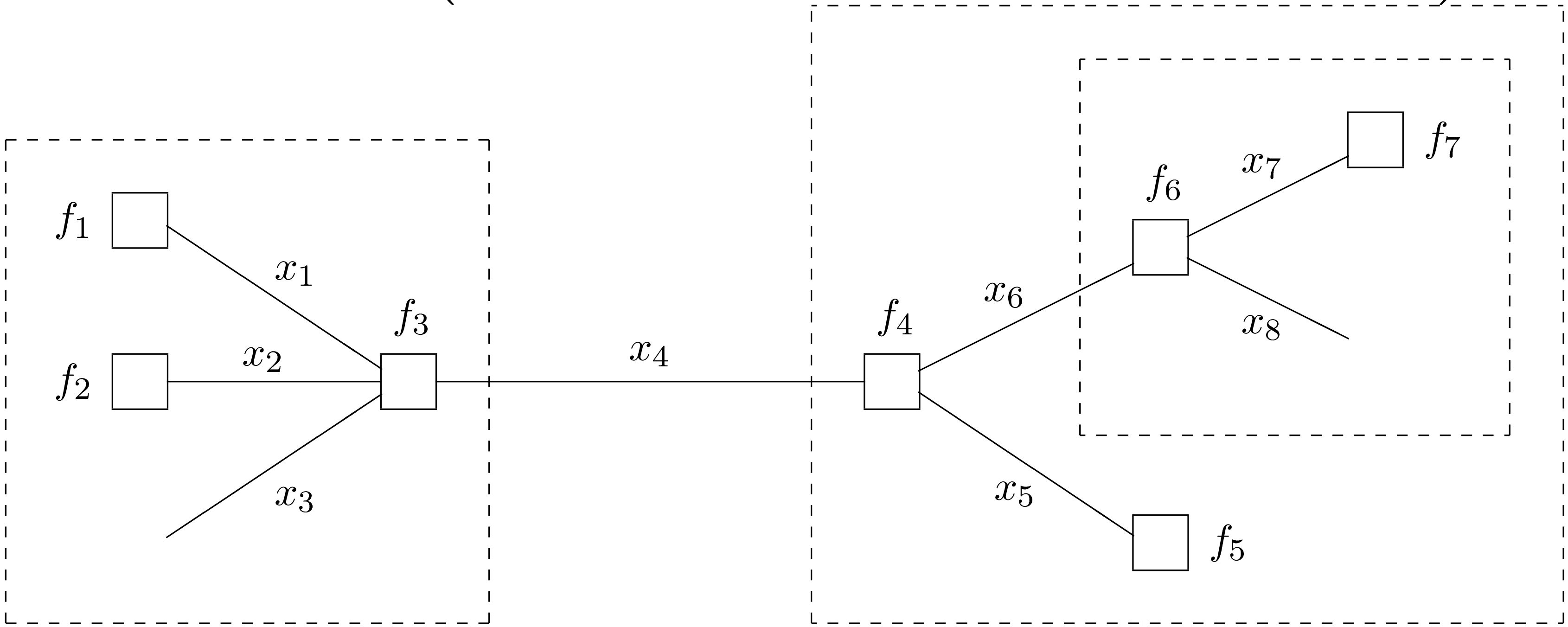
- Wanted: Marginal probability

$$p(x_4) = \sum_{x_1, x_2, x_3, x_5, x_6, x_7, x_8} f(x_1, \ldots, x_8)$$

- This factorization can be represented by a factor graph.

Elimination of Variables

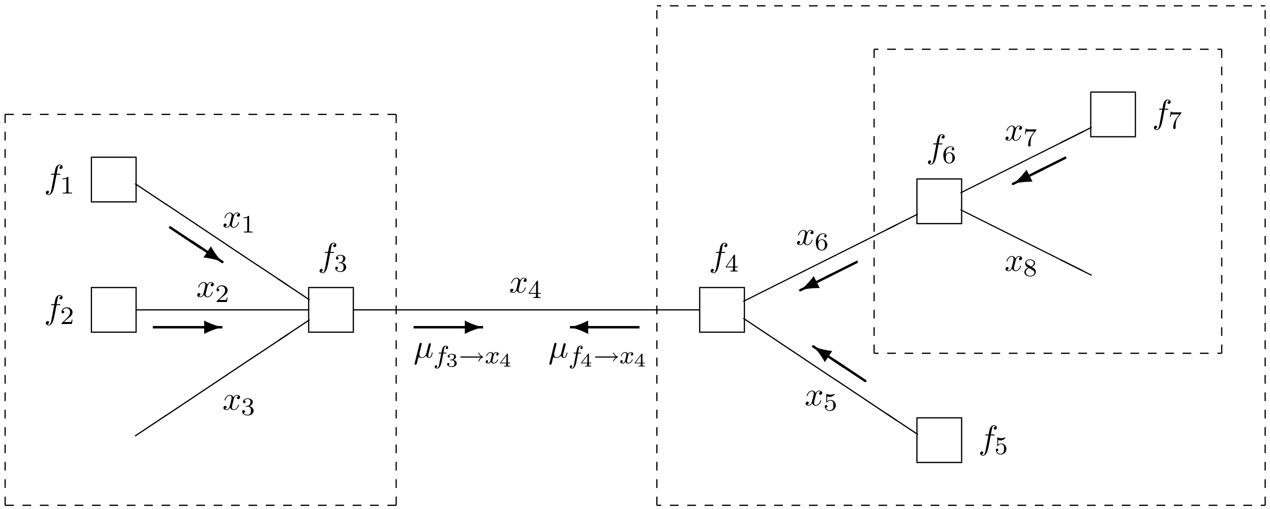
$$f(x_1, ..., x_8) \quad = \quad \big(f_1(x_1) f_2(x_2) f_3(x_1, x_2, x_3, x_4)\big) \cdot$$
$$\Big(f_4(x_4, x_5, x_6) f_5(x_5) \big(f_6(x_6, x_7, x_8) f_7(x_7)\big)\Big)$$



Elimination of Variables

$$p(x_4) \quad = \quad \underbrace{\left(\sum_{x_1} \sum_{x_2} \sum_{x_3} f_3(x_1, x_2, x_3, x_4) f_1(x_1) f_2(x_2) \right)}_{\mu_{f_3 \to x_4}} \cdot$$

$$\underbrace{\left(\sum_{x_5} \sum_{x_6} f_4(x_4, x_5, x_6) f_5(x_5) \underbrace{\left(\sum_{x_7} \sum_{x_8} f_6(x_6, x_7, x_8) f_7(x_7) \right)}_{\mu_{f_6 \to x_6}} \right)}_{\mu_{f_4 \to x_4}}$$



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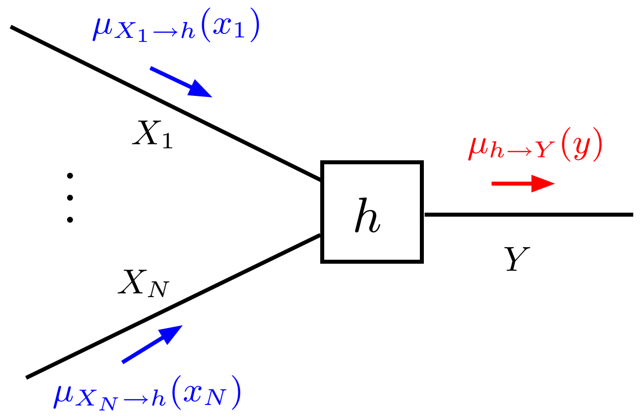
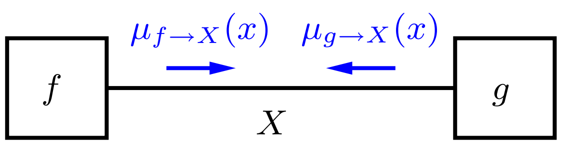
Sum-product Algorithm

Sum-product update rule



$$\mu_{h\to Y}(y) = \sum_{x_1,\ldots,x_N} h(x_1,\ldots,x_N,y)\mu_{X_1\to h}(x_1)\ldots\mu_{X_N\to h}(x_N).$$

Computing marginals



$$p(X) = \gamma\mu_{f\to X}(x)\mu_{g\to X}(x)$$

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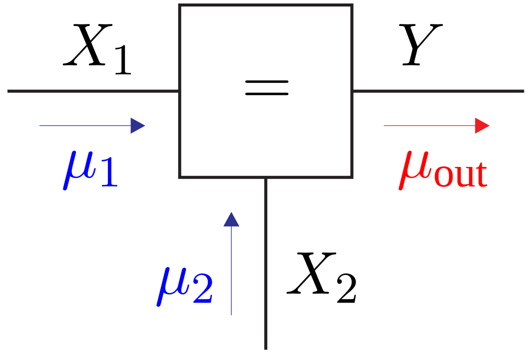
Sum-product Algorithm: Equal Node

$$f_=(x_1, x_2, y) \triangleq \delta[x_1 - x_2] \cdot \delta[x_1 - y]$$



For binary variables X_1, X_2, and Y we get:

$$\begin{aligned}
\mu_{\text{out}}(y) &= \sum_{x_1} \sum_{x_2} \delta[x_1 - x_2]\delta[x_1 - y]\mu_1(x_1)\mu_2(x_2) \\
&= \mu_1(y)\mu_2(y) \\
\mu_{\text{out}}(0) &= \mu_1(0)\mu_2(0) \\
\mu_{\text{out}}(1) &= \mu_1(1)\mu_2(1)
\end{aligned}$$

Sum-product Algorithm

- Factor graph without loops
  - The sum-product algorithm computes all marginals simultaneously and exactly.
- Factor graph with loops
  - The sum-product rule can be applied repeatedly: iterative algorithm.
  - The iterative algorithm might not converge.
    - Then another factor graph (factorization) can be tried.
    - There are several factor graph design choices.
  - The algorithm computes only an approximation of the marginals $p(x_k)$.
  - This approximation is often (but not always!) good enough.
  - In some very demanding applications the performance is nearly optimal.
  - The algorithm is suboptimal - but it can handle very large systems.

Intermediate Summary

- A factor graph is a graphical representation of a mathematical model.

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- The sum-product algorithm
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  - is suboptimal on graphs with cycles.
- However, iterative algorithms on graphs with cycles have been proven to be very powerful.
- Applications of iterative algorithms on graphs are, e.g.,
  - Decoding of error correcting codes,
  - Signal processing.

Overview

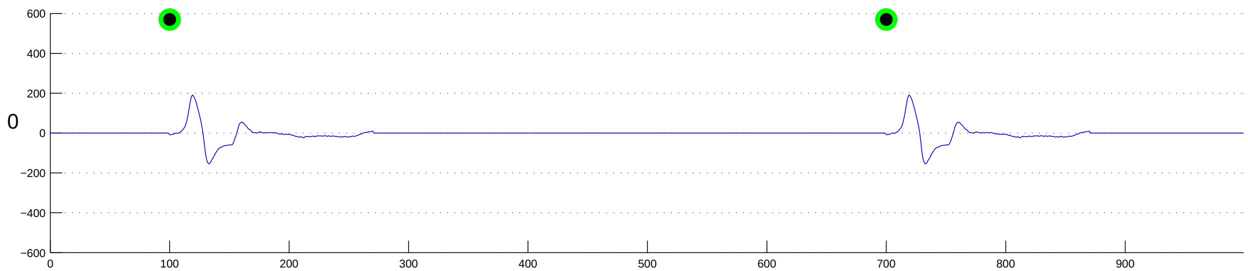
- Factor graphs
- Sum-product algorithm
- Application: decomposition of EMG-signals
- Conclusions

EMG Signal Decomposition

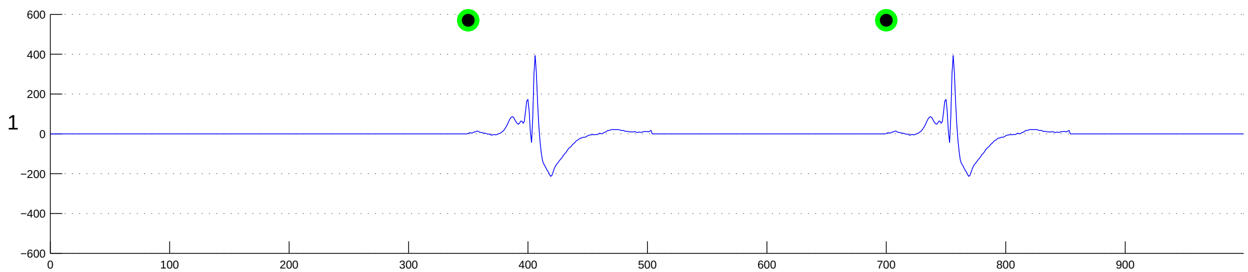
• = actual firing

● = detected firing

Signal from MU 0:

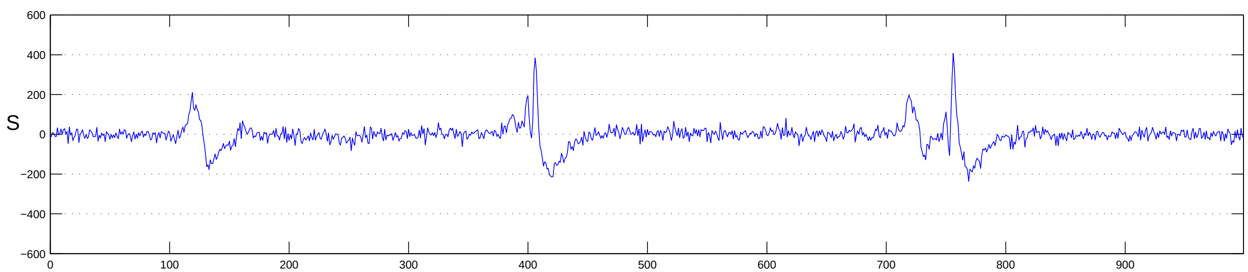


Signal from MU 1:



EMG signal:

$(\sigma_{\text{noise}} = 20)$

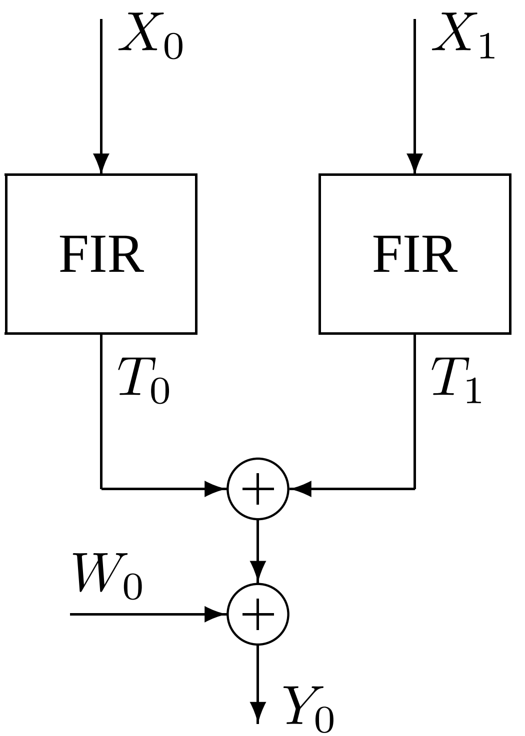


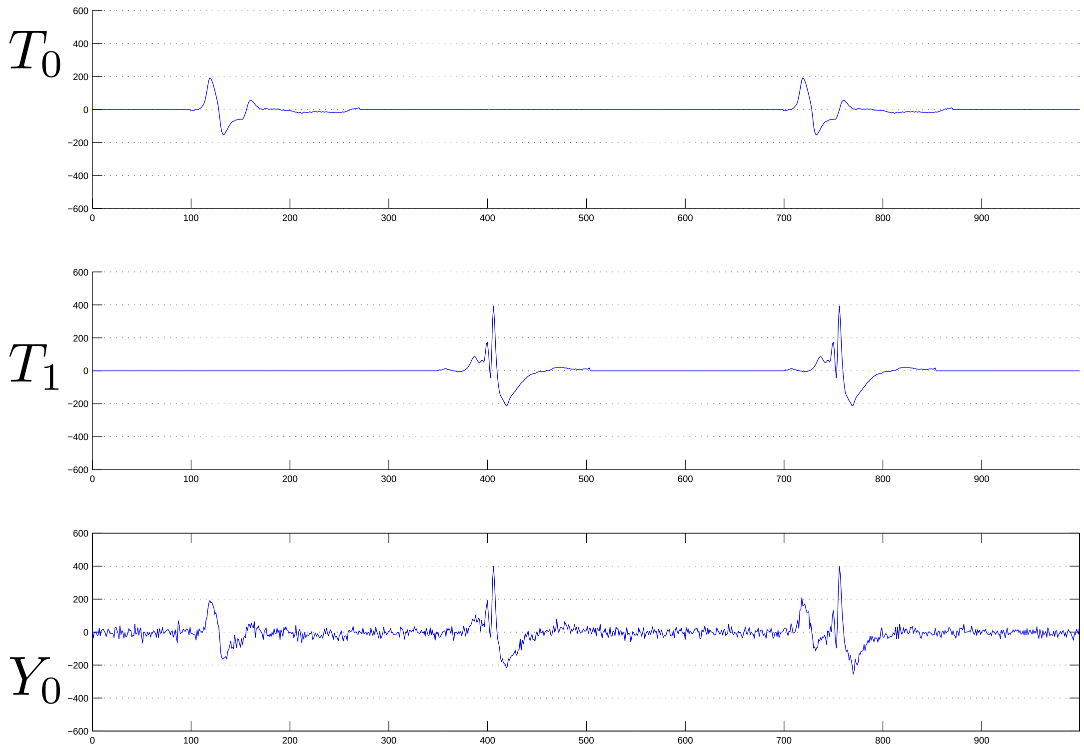
Lab History and Objective

- Our lab: 8 PhD students in the field of EMG since 1975
- *1992: Richard Gut (Prof. Moschytz)*
- *2000: Peter Wellig (Prof. Moschytz)*
- *200?: Volker Koch (Prof. Loeliger)*
  - Decomposition algorithm based on a
    - message-passing algorithm (belief propagation) in a
    - graphical model (factor graph)
  - *Preliminary result*: decomposition of more than 8 overlapping MUAPs
  - Information on the web
    - Volker's Webpage: `http://www.volker-koch.de/`
    - EMG forum: `http://www.emg.ethz.ch`

Block Diagram → Factor Graph

Block Diagram



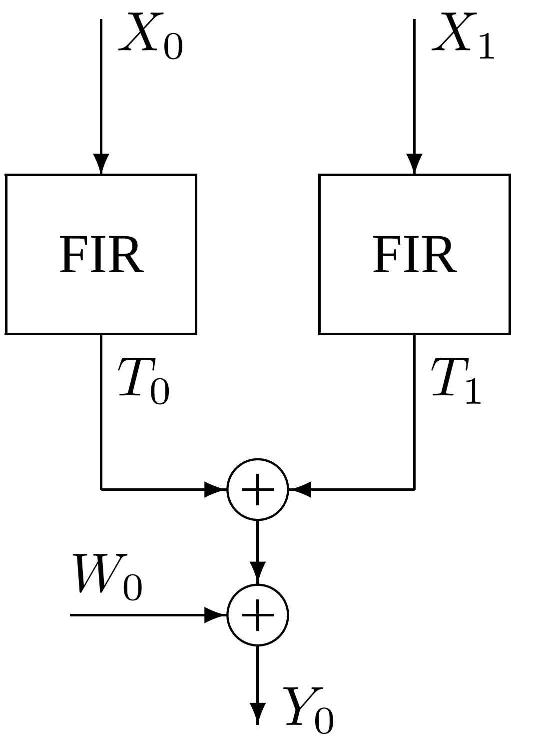


$$Y_{0,k} = \sum_{i=0}^{1} \sum_{\ell=0}^{M} X_{i,k-\ell} \cdot h_{i,0,\ell} + W_{0,k}$$

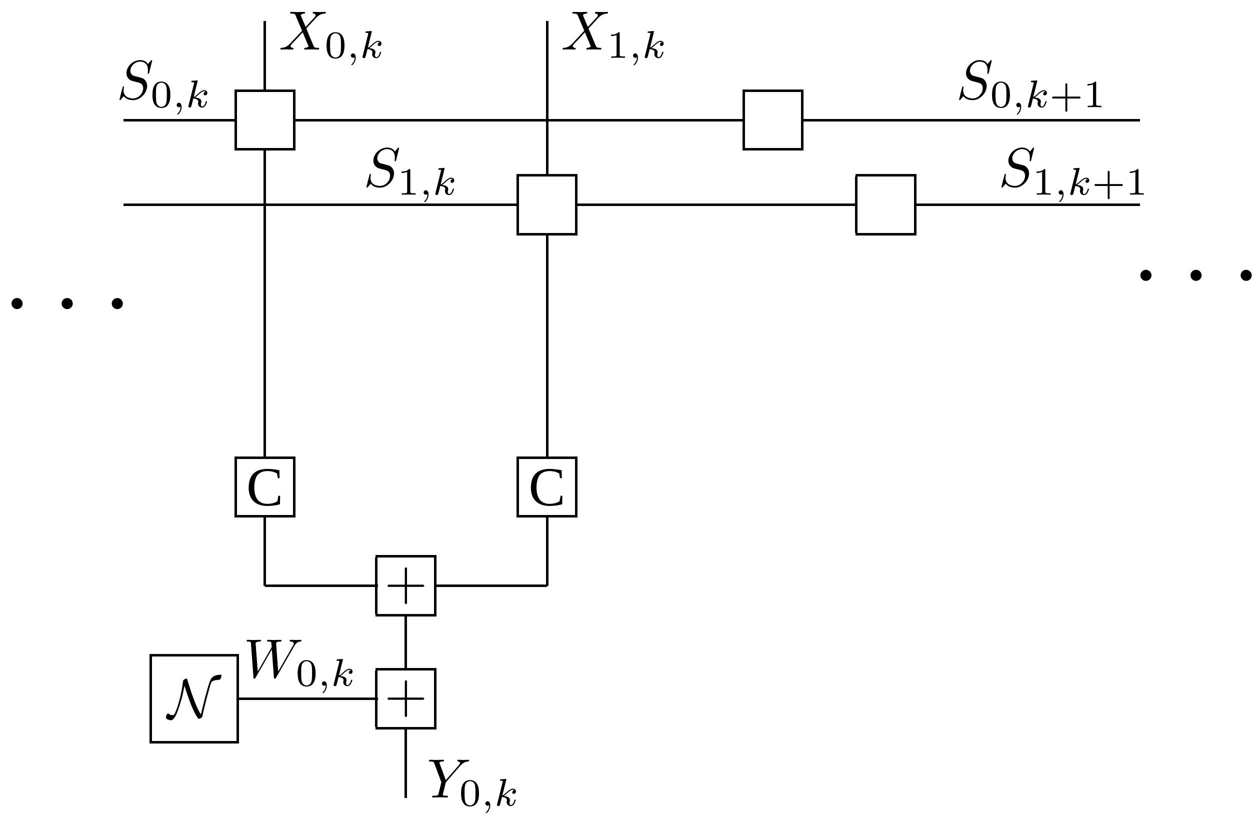
$$Y_0 \longrightarrow X_0, X_1 \quad ?$$

Block Diagram → Factor Graph

Block Diagram



Factor Graph



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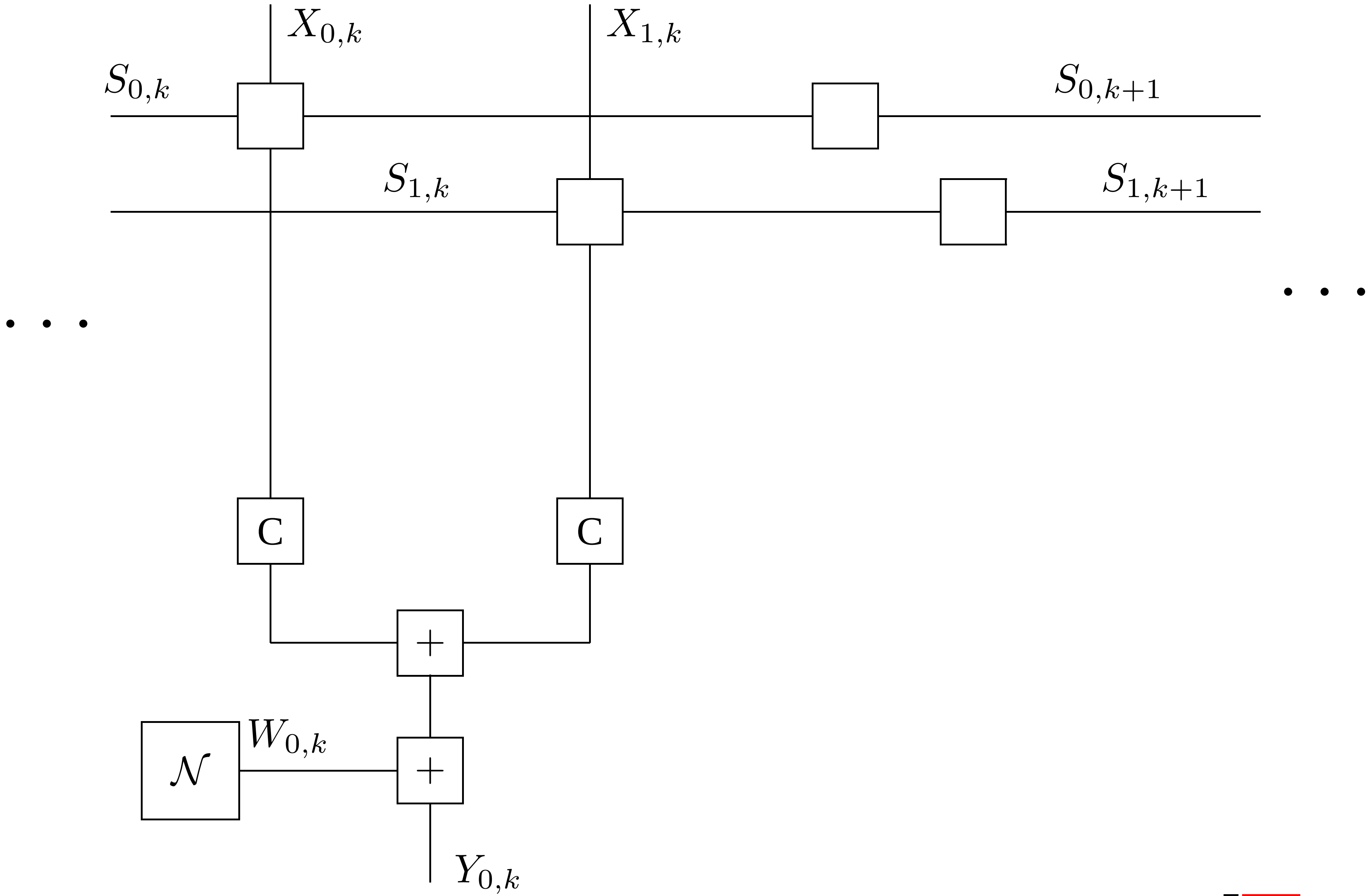
$$Y_0 \rightarrow X_0, X_1 \quad \textbf{?}$$

Estimating Motor Unit Firing Times



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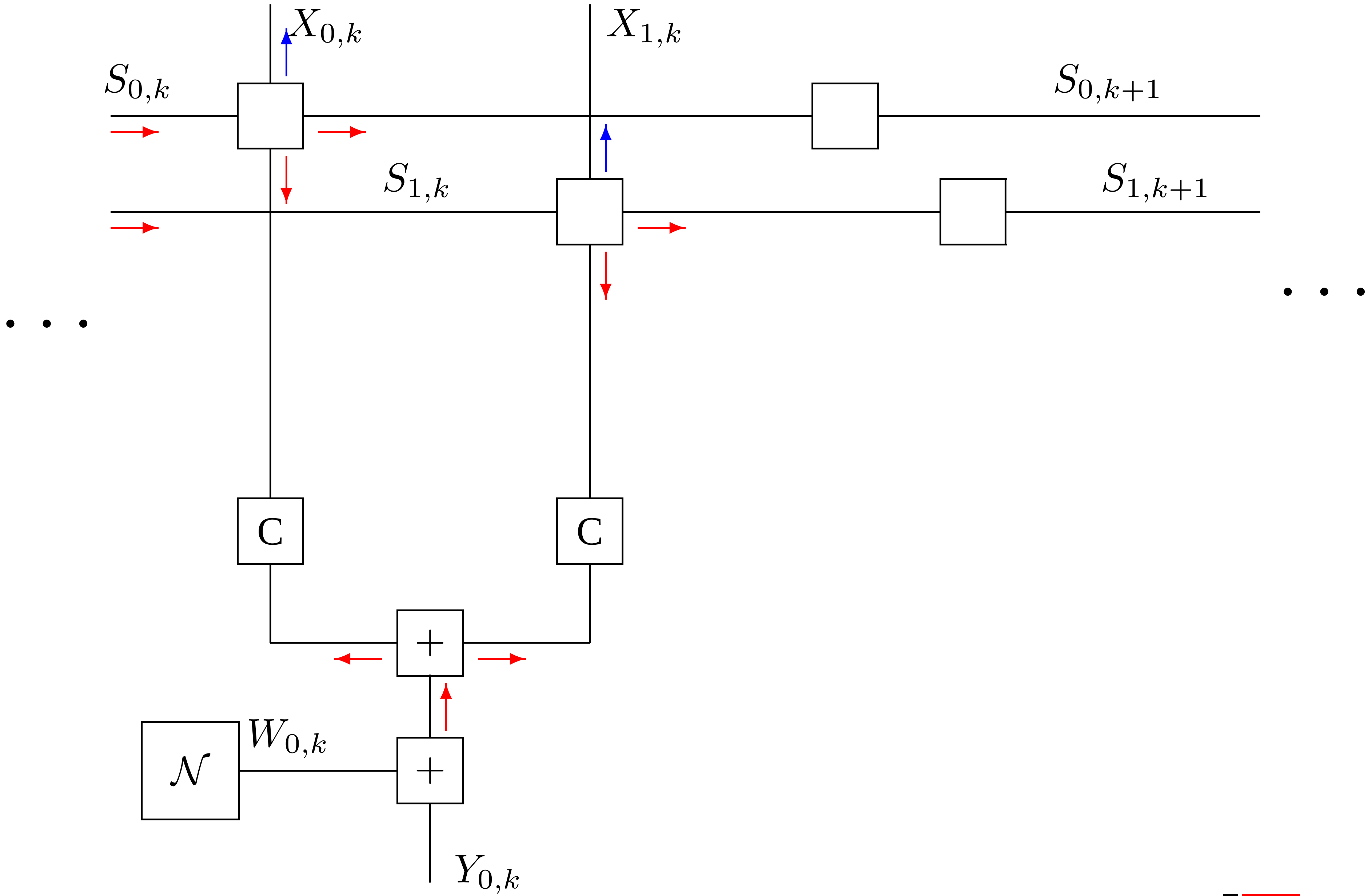
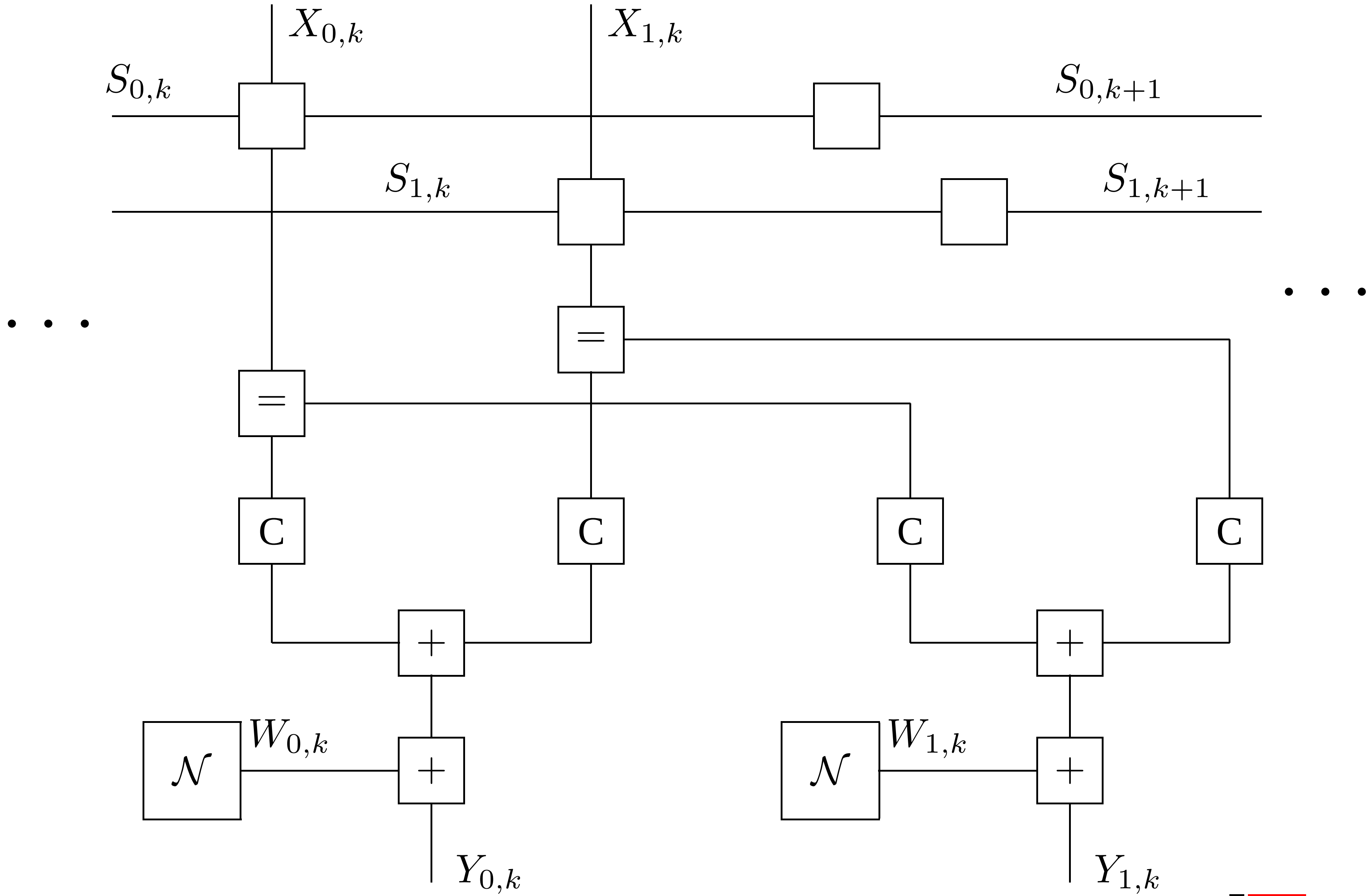
Estimating Motor Unit Firing Times



Factor Graph for Two Channels



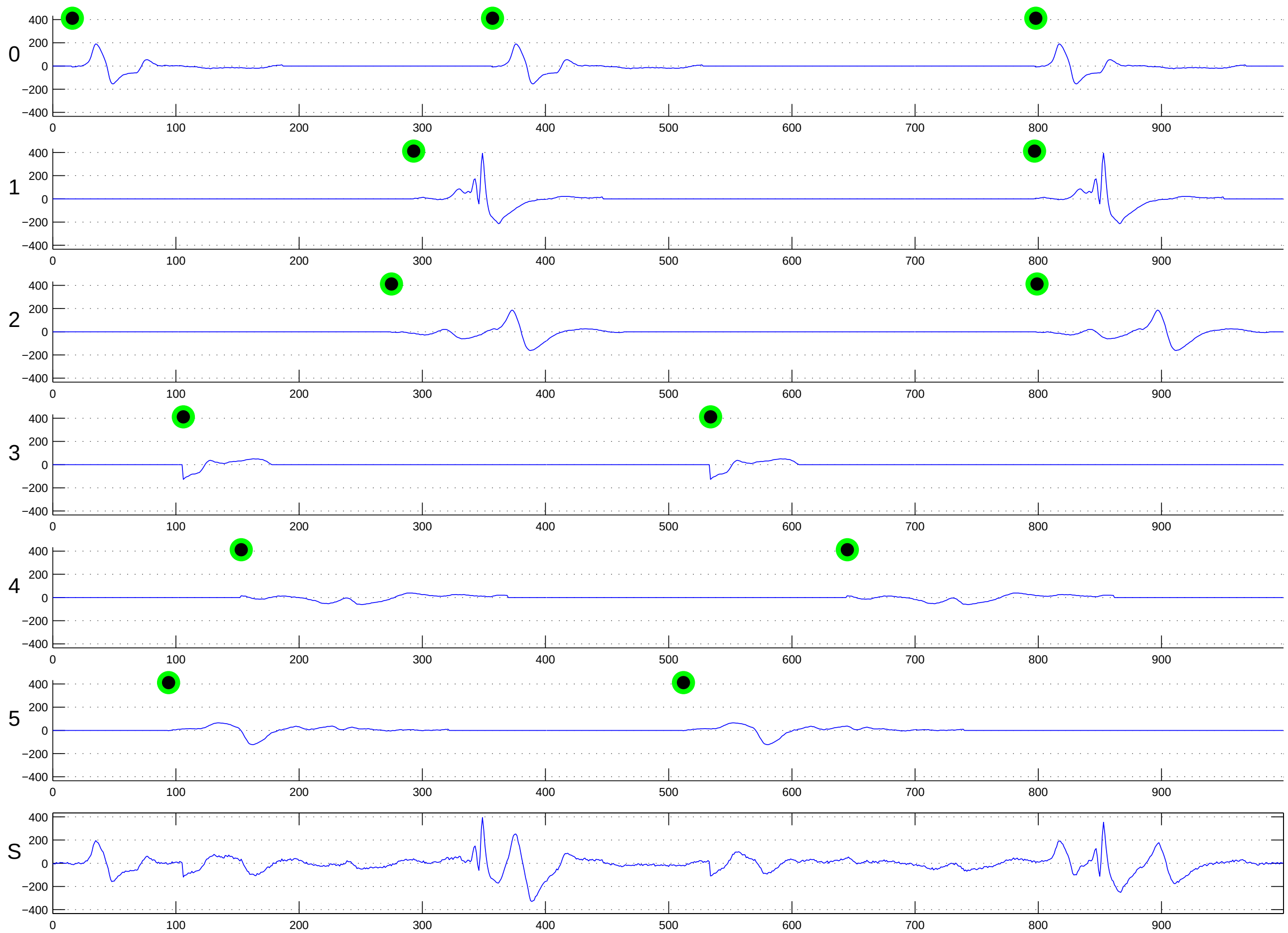
Preliminary Results

- **Simulated signals**
  - Decomposition of superpositions of more than 8 overlapping MUAPs
- **Measured signals**
  - First tests done
- **Changing MUAP shapes in simulated long-term recordings**
  - First tests done
- **MUAP shapes**
  - Need to be given to our algorithm
- **Noise model**
  - AWGN, tried others

Results: 1 Simulated Signal, 6 MU

• = actual firing

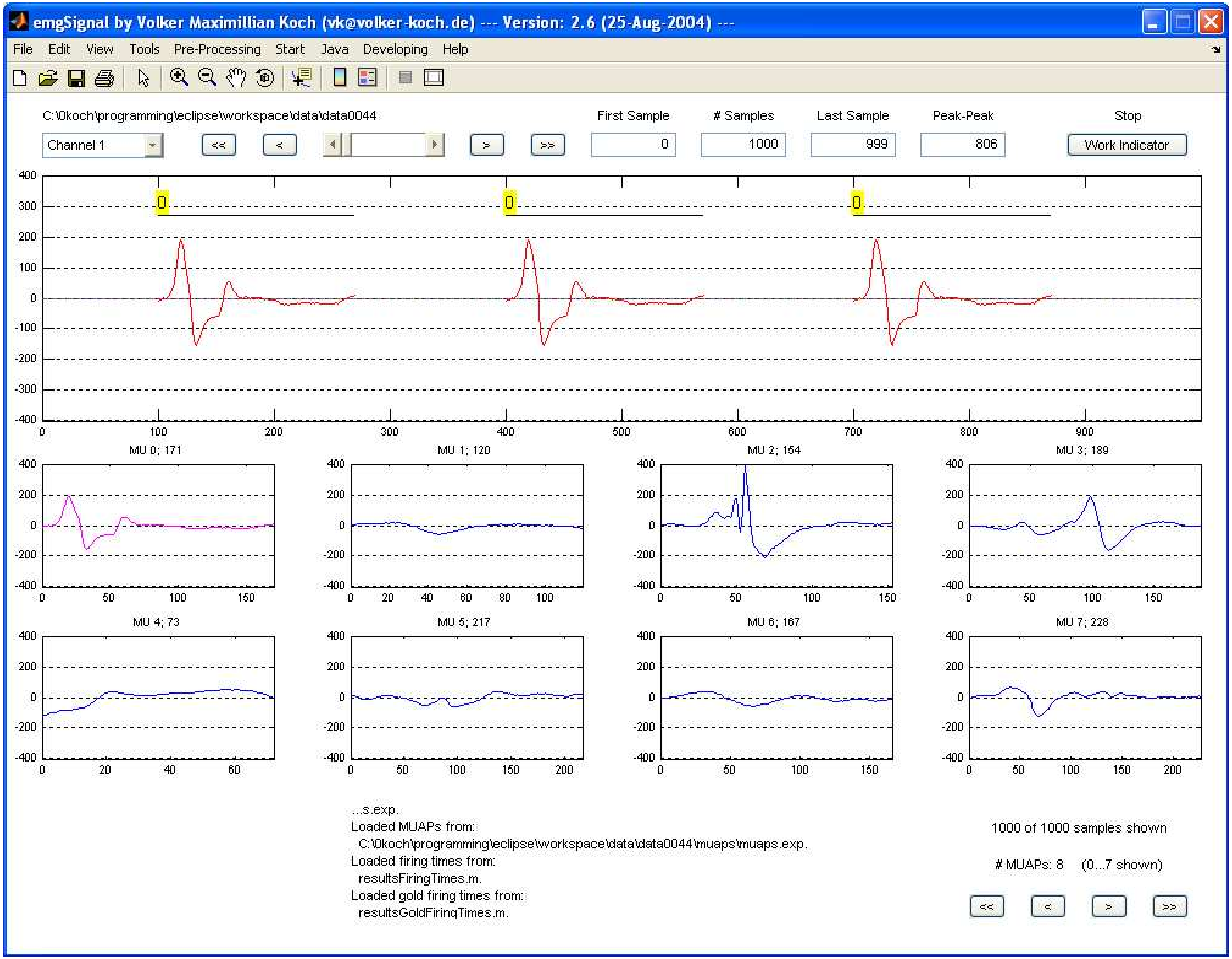
● = detected firing



Results: 3000 Simulated Signals, 6 MU

σ_{noise}	Number of completely correct decomposed signals / total number of signals
0	$899/1000$
5	$836/1000$
10	$720/1000$

Results: 1 Simulated Signal, 8 MU



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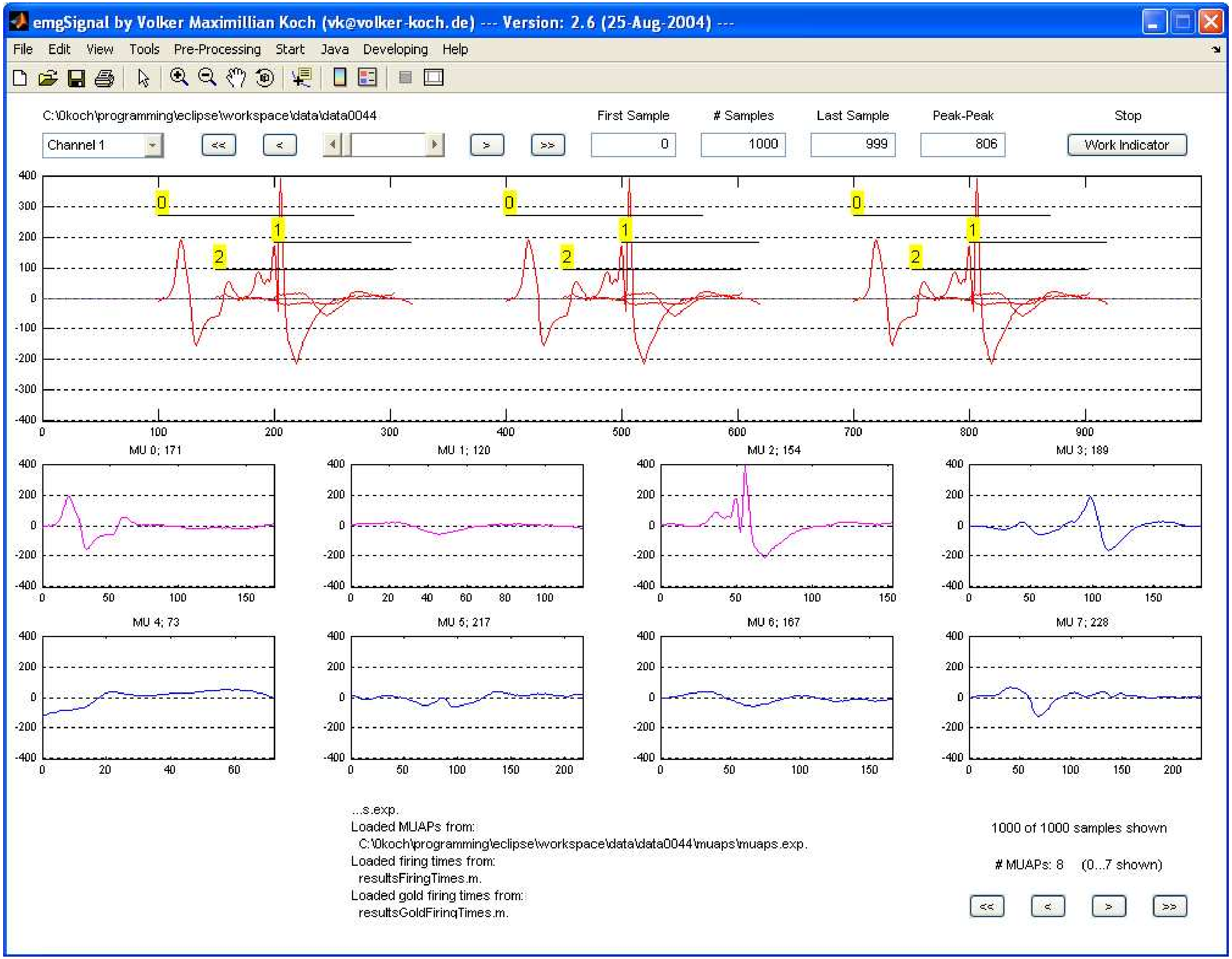

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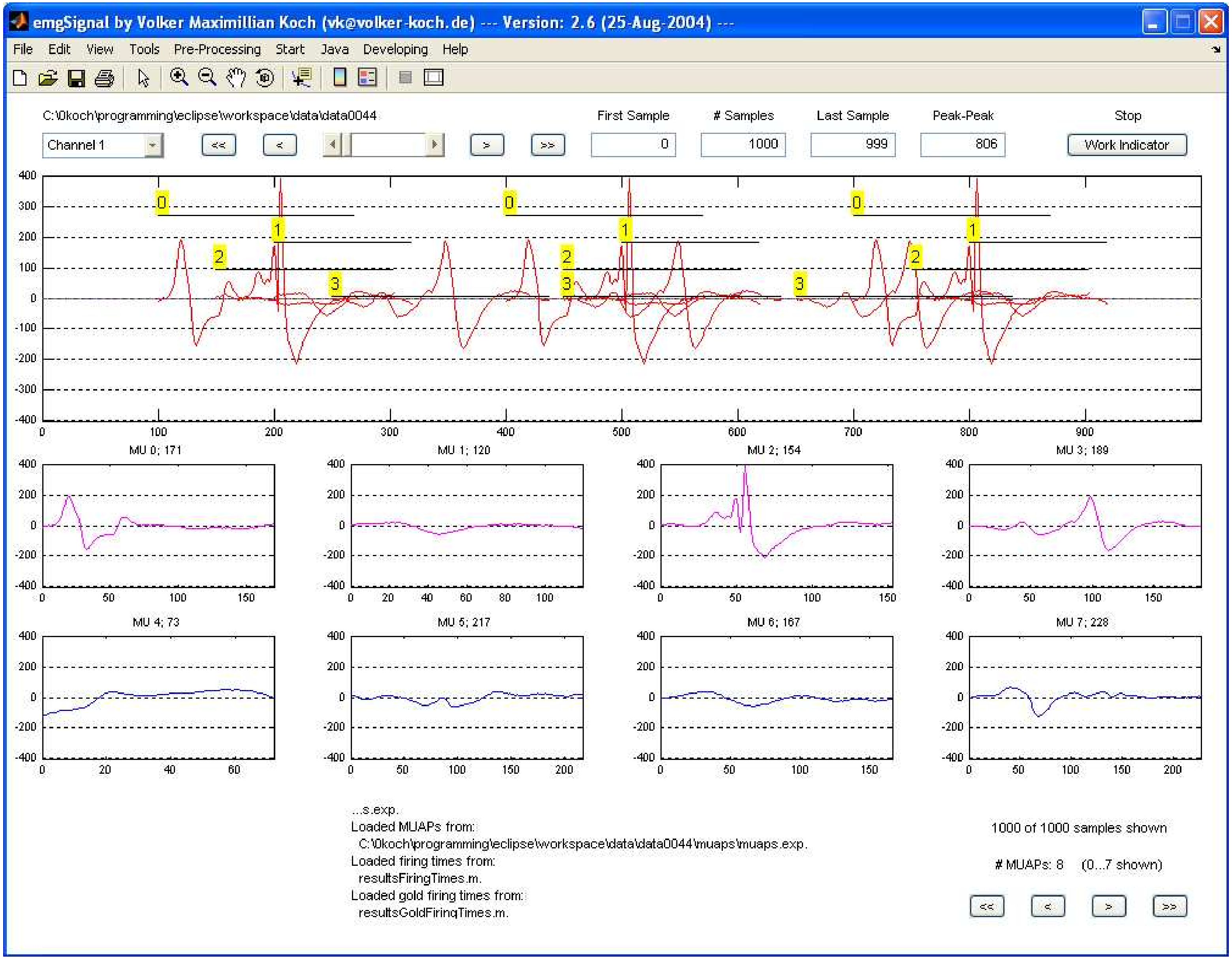

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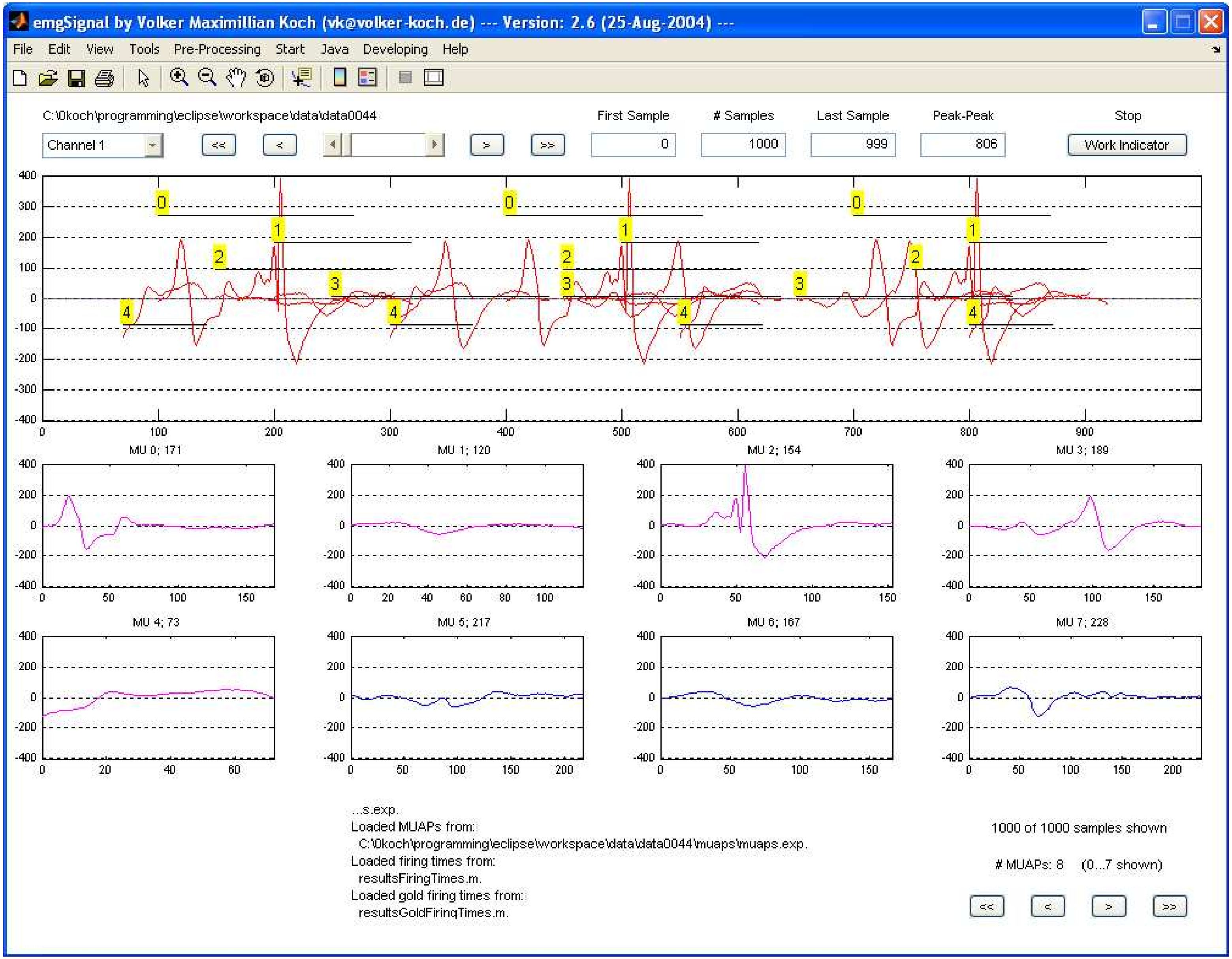

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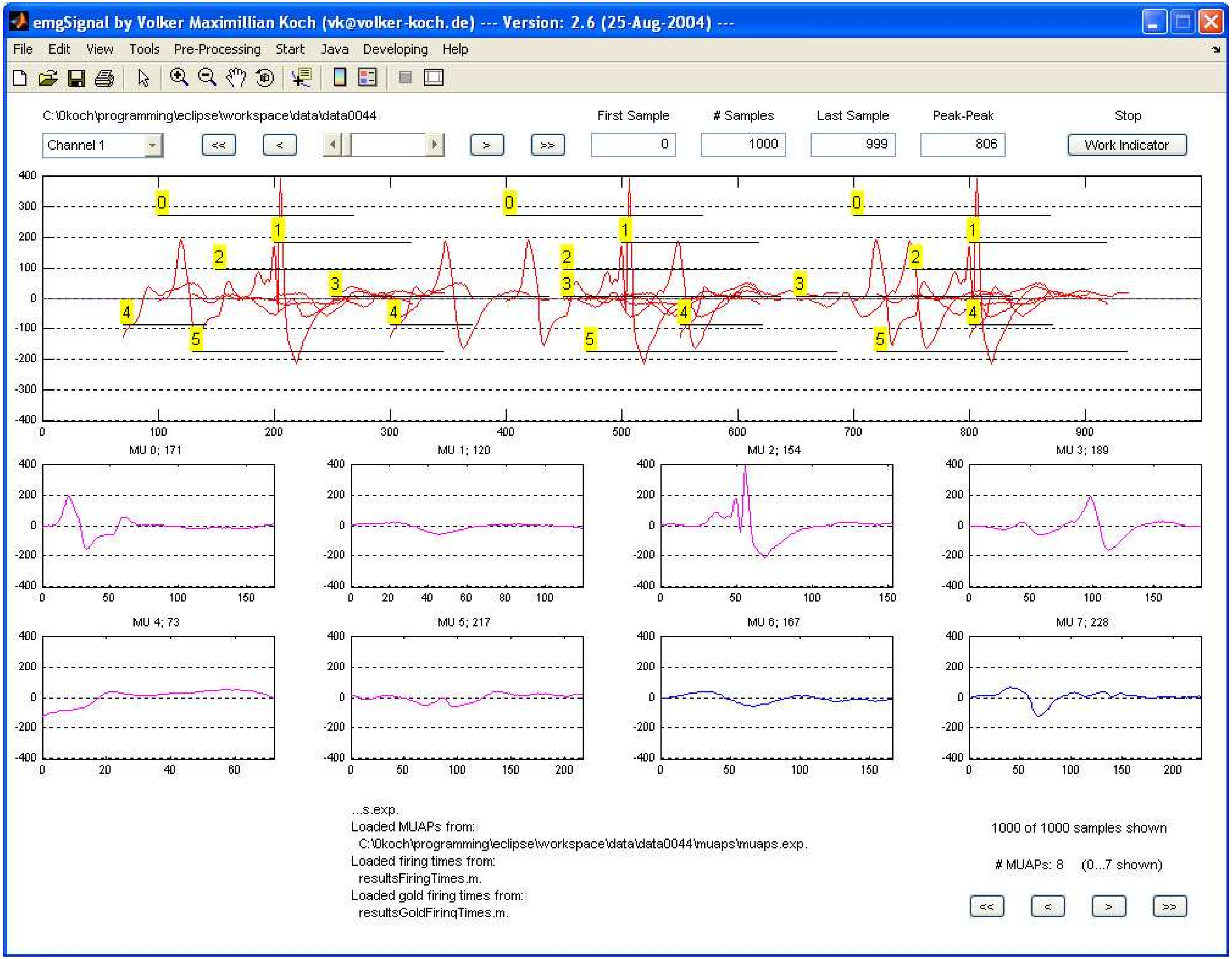
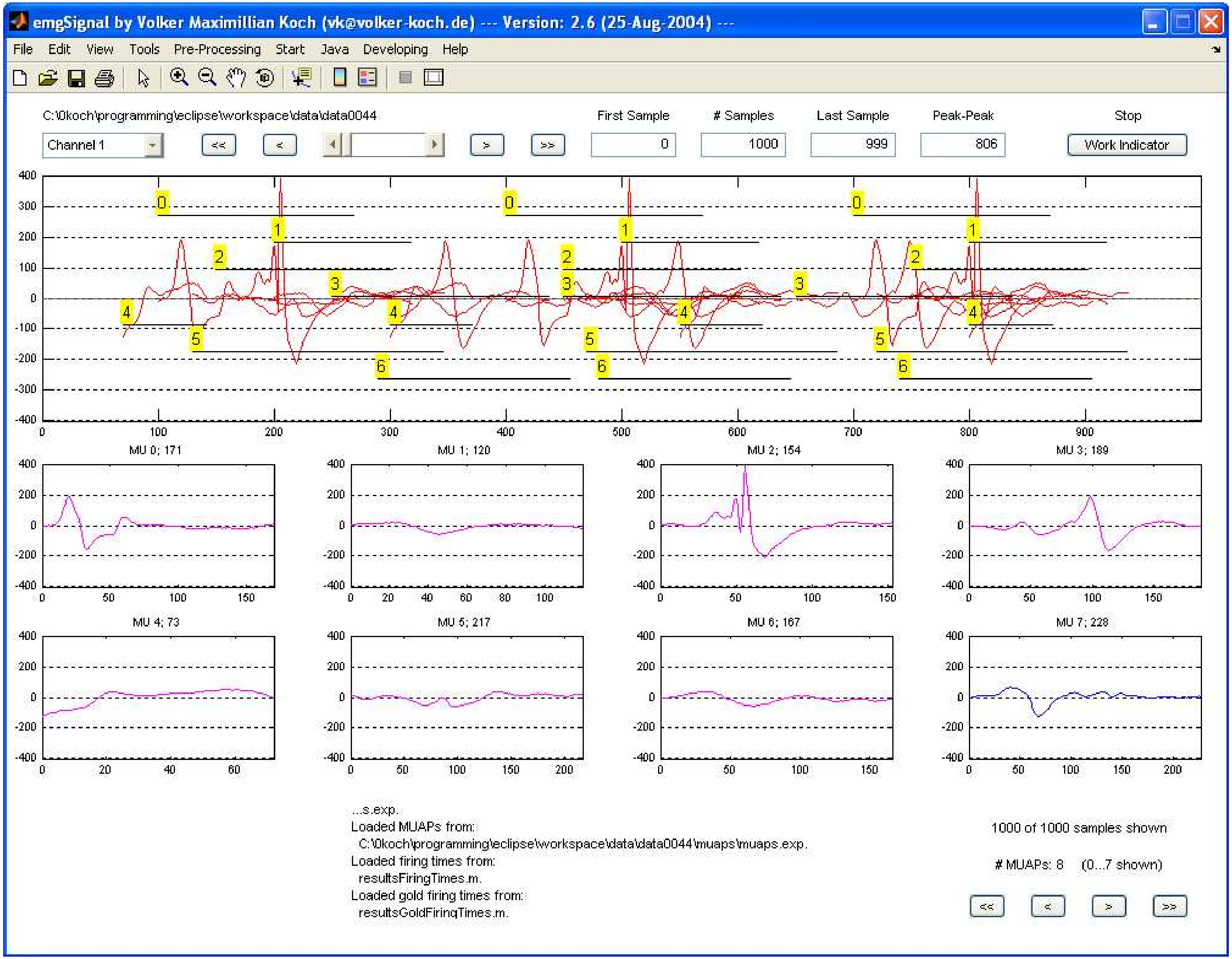


Results: 1 Simulated Signal, 8 MU



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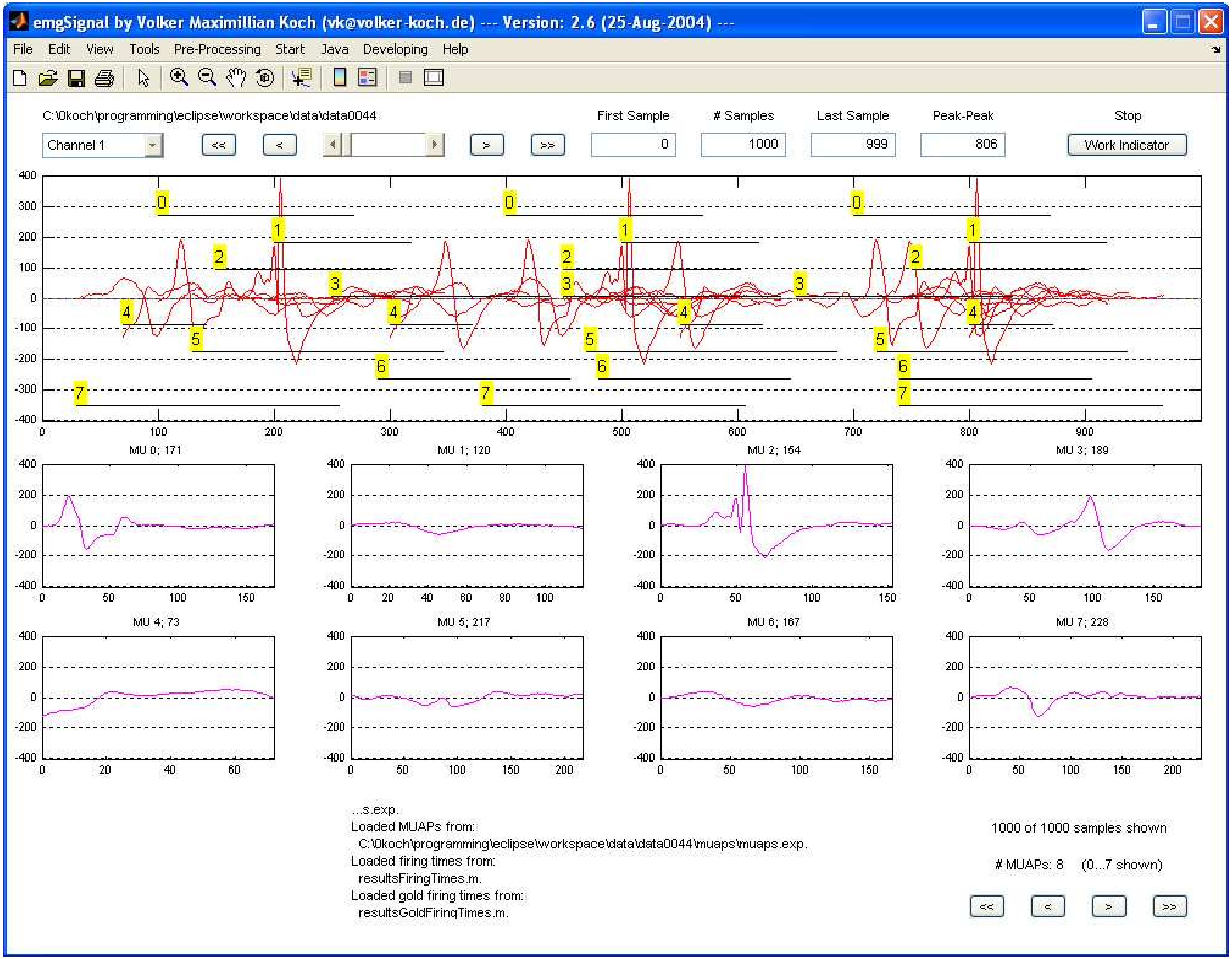
Results: 1 Simulated Signal, 8 MU



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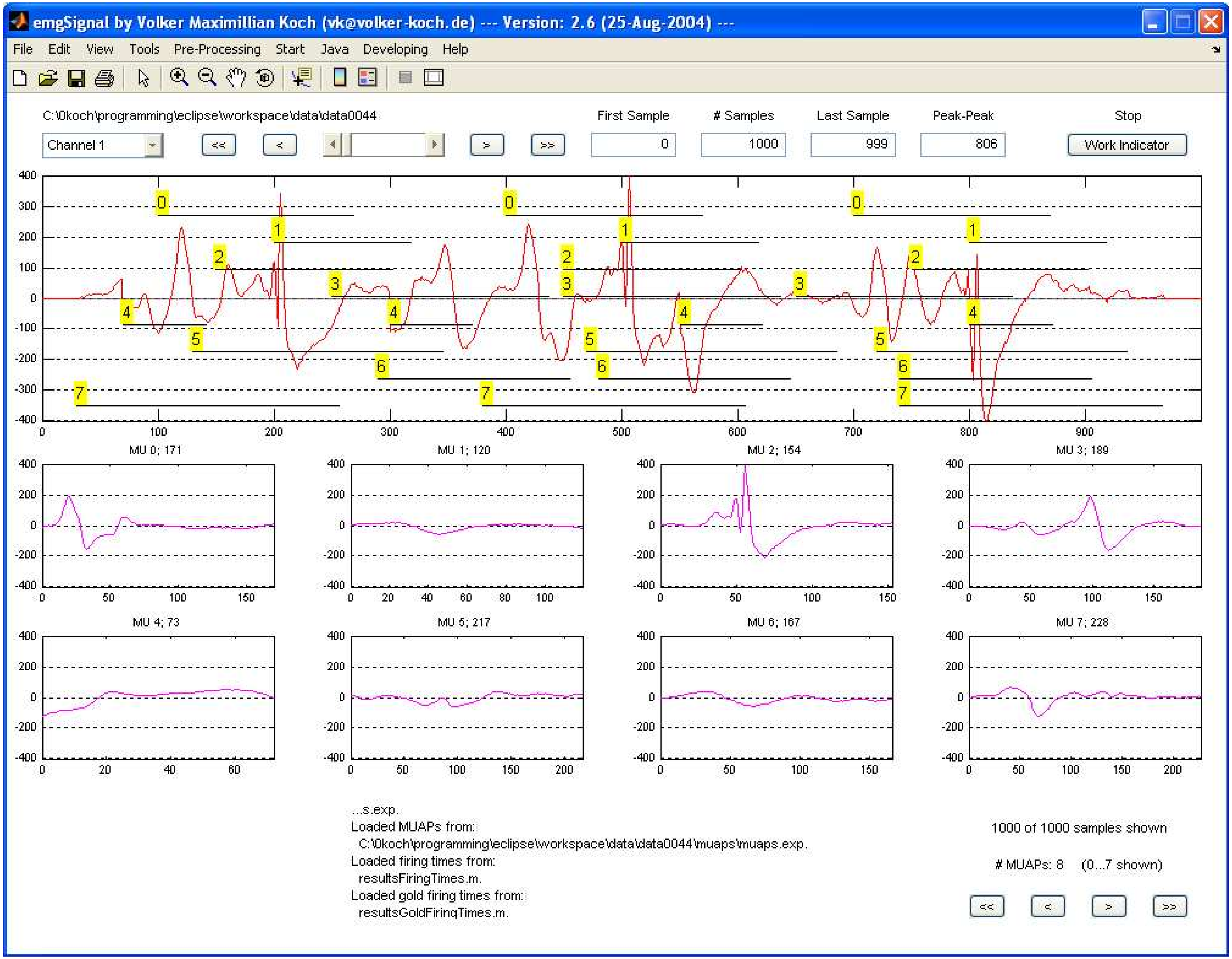
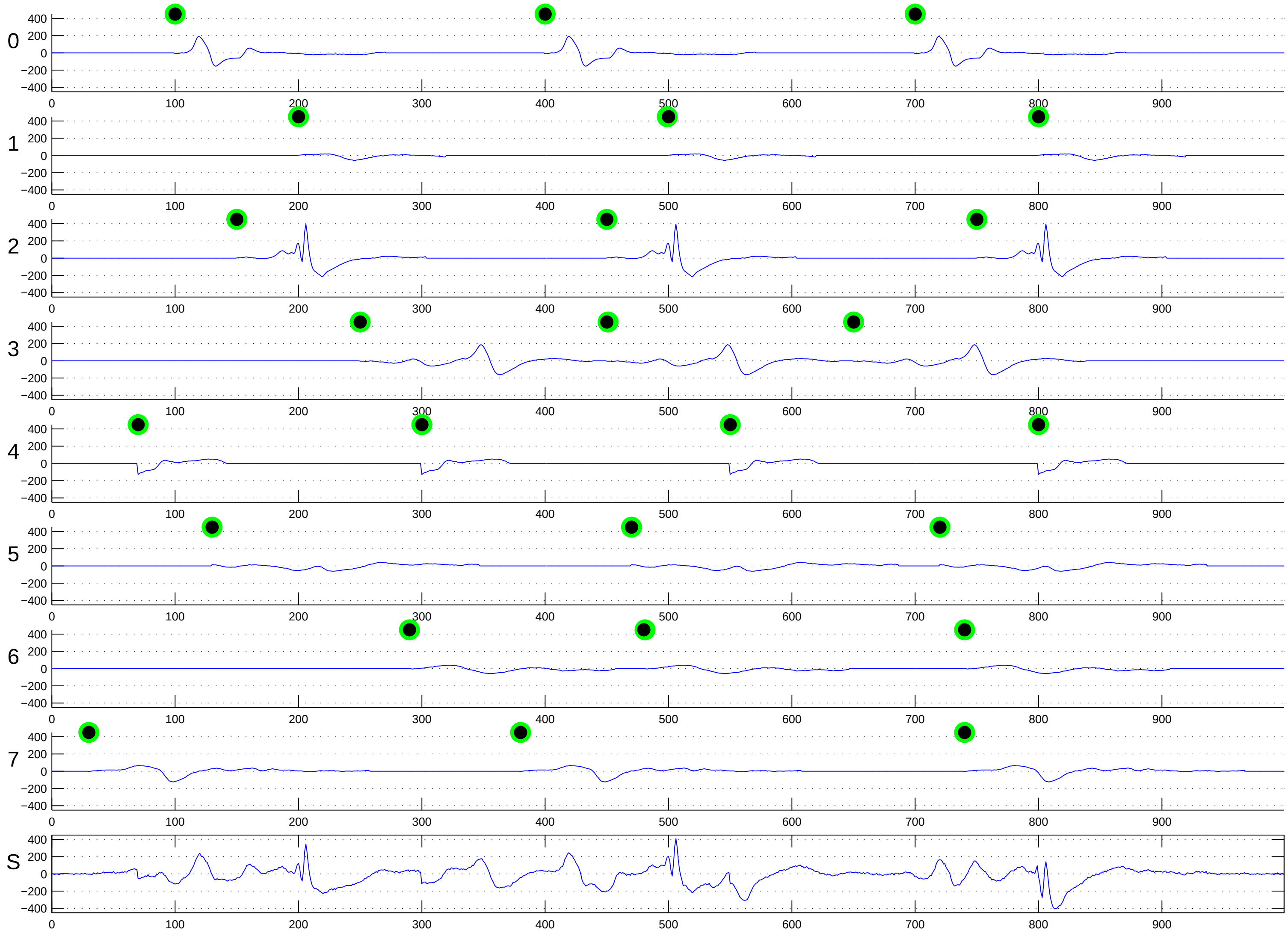
Swiss Federal Institute of Technology Zurich

Signal and Information Processing Laboratory
Institut für Signal- und Informationsverarbeitung

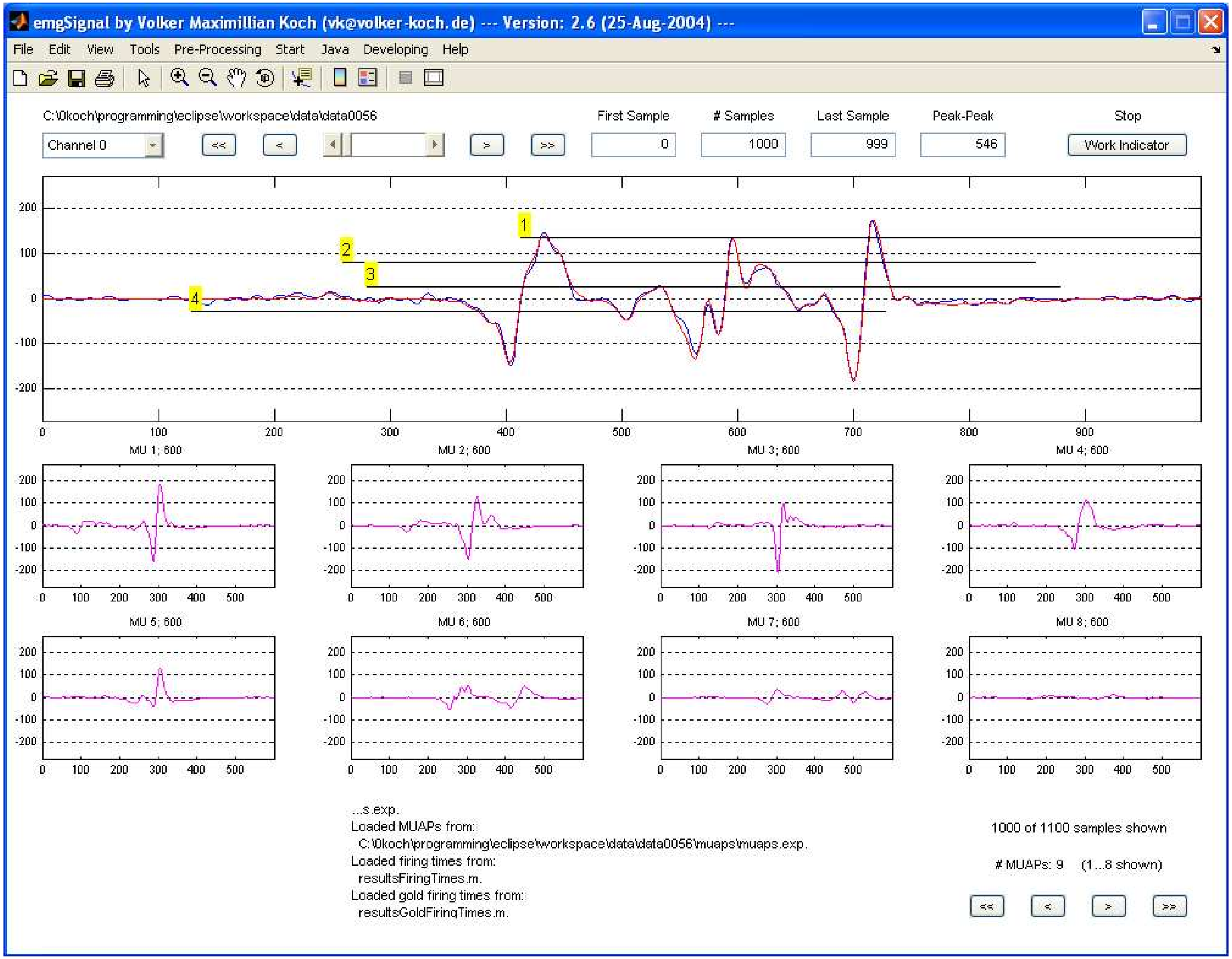
Results: 1 Simulated Signal, 8 MU



Results: 1 Simulated Signal, 8 MU



Results: 1 Measured Signal, 9 MU



Thanks to Kevin McGill for providing the EMG signal!

Overview

- Factor graphs
- Sum-product algorithm
- Application: decomposition of EMG-signals
- Conclusions

Conclusions

- **EMG signal decomposition algorithm**
- **Based on belief propagation in a graph**
- **Aim: Resolving difficult superpositions without much a-priori information**
  - e.g., no firing statistics
  - e.g., dynamic contractions
  - e.g., doublets

Applications of graphical models in signal processing

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